Readout of quasi-periodic systems using qubits

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Abstract

We develop a theoretical scheme to perform a readout of the properties of a quasiperiodic system by coupling it to one or two qubits. We show that the decoherence dynamics of a single qubit coupled via a pure dephasing type term to a one-dimensional quasiperiodic system with a potential given by the AAH model and its GAAH model is sensitive to the nature of the single-particle eigenstates(SPEs). More specifically, we can use the non-Markovianity of the qubit dynamics as quantified by the backflow of information to clearly distinguish the localized, delocalized, and mixed regimes with a mobility edge ofthe AAH and GAAH models and evidence the transition between them. By attaching two qubits at distinct sites of the system, we demonstrate that the transport property of the quasiperiodic system is encoded in the scaling of the threshold time to develop correlations between the qubits with the distance between the qubits.

Motivation

- Quasiperiodic systems have rich localization and transport properties even in 1D. One of these systems is AAH potential AAH potential, where by tuning the system parameters it is possible to get delocalized, critical, and localized single particle states. Moreover, various generalized versions of the AAH potentials that support a single-particle mobility edge even in 1D have been proposed.
- The intense theoretical research in this area focusing on different properties such as single-particle and many body localization, transport, etc. has been complemented and invigorated by the experimental realizations of such quasiperiodic potentials and the detection of localization and other associated phenomena [1,2].
- The open and isolated quasi-periodic systems have been studied in various manner [1]. Our aim is to take a middle path where we willcouple the quasiperiodic system to qubits, thereby in a controlled manner "opening" the system with the goal of inferring the localization and transport properties of the system from the dynamics of the qubits.

Main Results

1. We readout the properties of quasi-periodic systems via studying the decoherence dynamics of one qubit and correlation dynamics of two qubits.
2. For the readout purpose we have considered pure dephasing spin-boson model.
3. The decoherence dynamics of single qubit is greatly affected by the properties of single-particle eigenstates. Thus studying the decoherence dynamics, we have found the delocalization-localization transition of AAH model and also we have confirmed the presence of mobility edge. Studying the correlation dynamics between two qubits, we have also extracted the isolated system transport properties of quasi-periodic systems. We have also found that in presence of mixture of all single-particle states (delocalized, critical and localized) states, transport is governed by the fastest spreading states.

Theory

- TB Hamiltonian for the bosonic system:

$$\hat{H} = \sum_{n=1}^{N} \left(\mu + \frac{2\lambda \cos[2\pi b n + \phi]}{1 + \alpha \cos[2\pi b n + \phi]} \right) \hat{c}_n^\dagger \hat{c}_n + \sum_{n=1}^{N-1} \left(\hat{c}_n^\dagger c_{n+1} + \hat{c}_{n+1}^\dagger \hat{c}_n \right) . \quad (1)$$

- Pure dephasing spin-boson model with one qubit:

$$\hat{H}_{1q} = \hat{H} + \frac{\omega_A}{2}\hat{\sigma}_z^i + g\hat{\sigma}_z^i(\hat{c}_i^\dagger + \hat{c}_i), \quad (2)$$

- Decoherence factor:

$$\langle \hat{\sigma}_-^i(t) \rangle = \langle \hat{\sigma}_-^i(0) \rangle e^{-i\omega_A t - \Gamma_i(t)}, \quad (3)$$

$$\Gamma_i(t) = 4\sum_{k=1}^{N} |g_k^i|^2 \coth\left(\frac{\beta\omega_k}{2}\right) \frac{1-\cos(\omega_k t)}{\omega_k^2}. \quad (4)$$

- Backflow of information:

$$\mathcal{N} = \sum_{p=1}^{N_{max}} (e^{-\Gamma(t_p^f)} - e^{-\Gamma(t_p^i)}), \quad (5)$$

- Pure dephasing spin-boson model with two qubits:

$$\hat{H}_{2q}^{SB} = \frac{\omega_A}{2}\hat{\sigma}_z^i + \frac{\omega_B}{2}\hat{\sigma}_z^j + \sum_{k=1}^{N} \omega_k \hat{\eta}_k^\dagger \hat{\eta}_k + \sum_{k=1}^{N} \hat{\sigma}_z^i (g_k^i \hat{\eta}_k^\dagger + g_k^{i*} \hat{\eta}_k) + \sum_{k=1}^{N} \hat{\sigma}_z^j (g_k^j \hat{\eta}_k^\dagger + g_k^{j*} \hat{\eta}_k) \quad (6)$$

- Isolated system transport properties: wave-packet width $w \sim t^\eta$. $\eta = 1 \rightarrow ballistic\ transport$, $\eta = 0.5 \rightarrow diffusive\ tranport$, $\eta = 0 \rightarrow no\ transport$

Results

0.1 One qubit coupled to quasi-periodic system

0.1.1 Prediction of on-site potential:

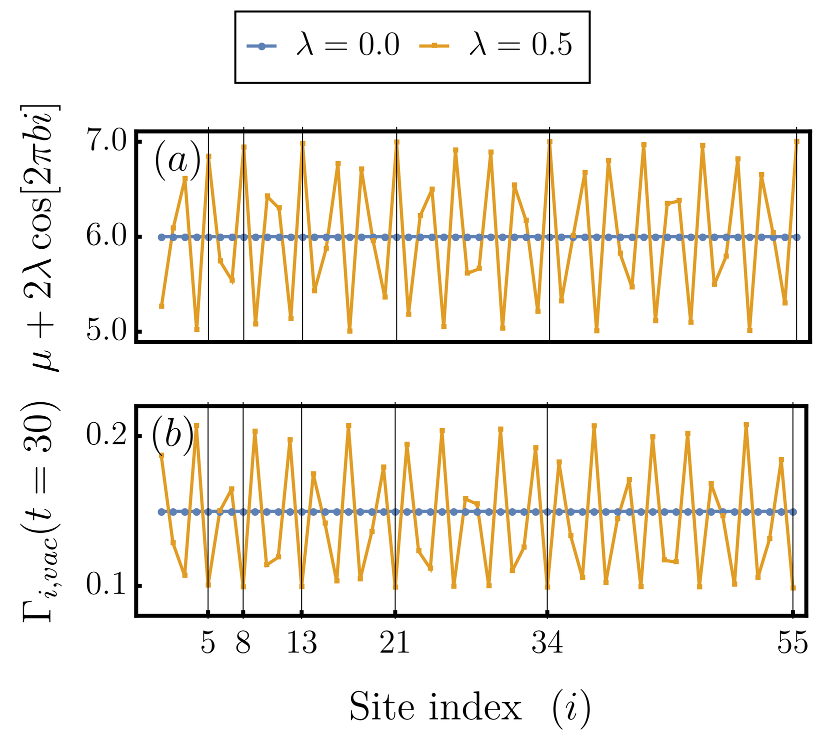

Figure 1: (a) Ordered ($\lambda = 0$, blue) and quasi-periodic AAH on-site potential ($\lambda = 0.5$, orange) (see Eq. (1)) as a function of site index (b). Vacuum decoherence factor for a single qubit as a function of the site to which it is coupled reflecting the underlying on-site potential (total number of sites = 55). Vertical lines denote the Fibbonacci numbers at which point the potential and decoherence factors almost repeat themselves, as expected for the irrational value $b = \frac{\sqrt{5}-1}{2}$.

0.1.2 Delocalization-localization transition:

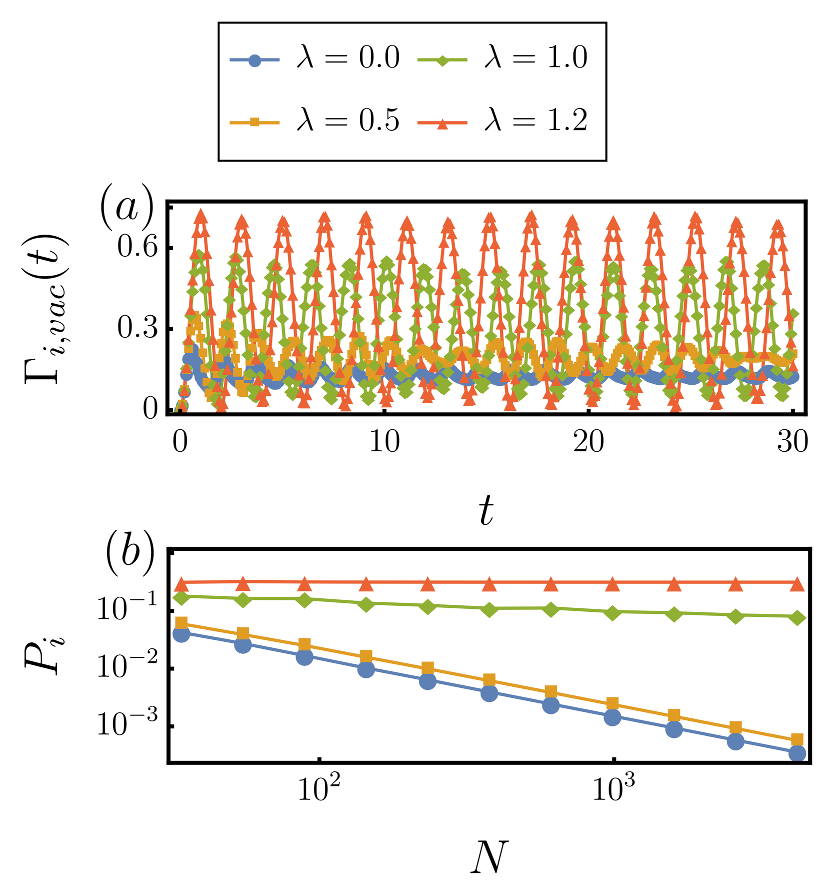

Figure 2: (a). Vacuum decoherence factor as a function of time for a 610-site regular AAH model ($\alpha = 0$) with the qubit coupled to site $i = 72$. The dynamics goes from damped to oscillatory as λ is tuned over delocalized ($\lambda = 0$ and $\lambda = 0.5$), critical (λ=1.0) and localized ($\lambda = 1.2$) regimes of the AAH model. (b). Inverse participation ratio of eigenmodes at the qubit coupling site i, P_i (see text for definition) as a function of the number of sites N. P_i scales as $1/N$ in the delocalized regime, N^0 for the localized case, and as N^{-b} with $0 < b < 1$ in the critical regime.

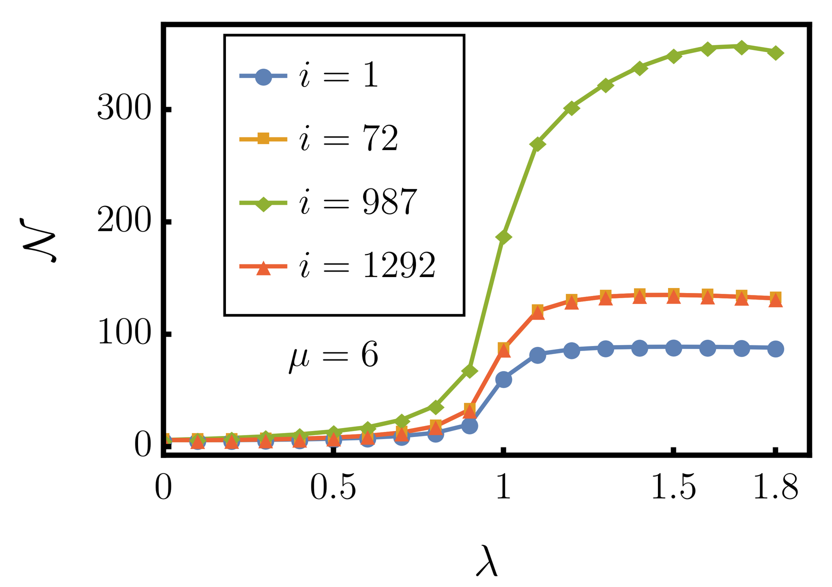

Figure 3: Backflow of information $\mathcal{N}$ measuring the non-markovianity in the decoherence dynamics of a qubit coupled to different sites (see legend) of a quasi-periodic lattice (regular AAH model $\alpha = 0$) as a function of λ (number of sites $N = 2584$). Clearly the non-markovianity measure is sensitive to the localization-delocalization transition at $\lambda = 1$.

0.2 Two qubit coupled to quasi-periodic systems and transport properties

0.2.1 Ballistic and diffusive transport:

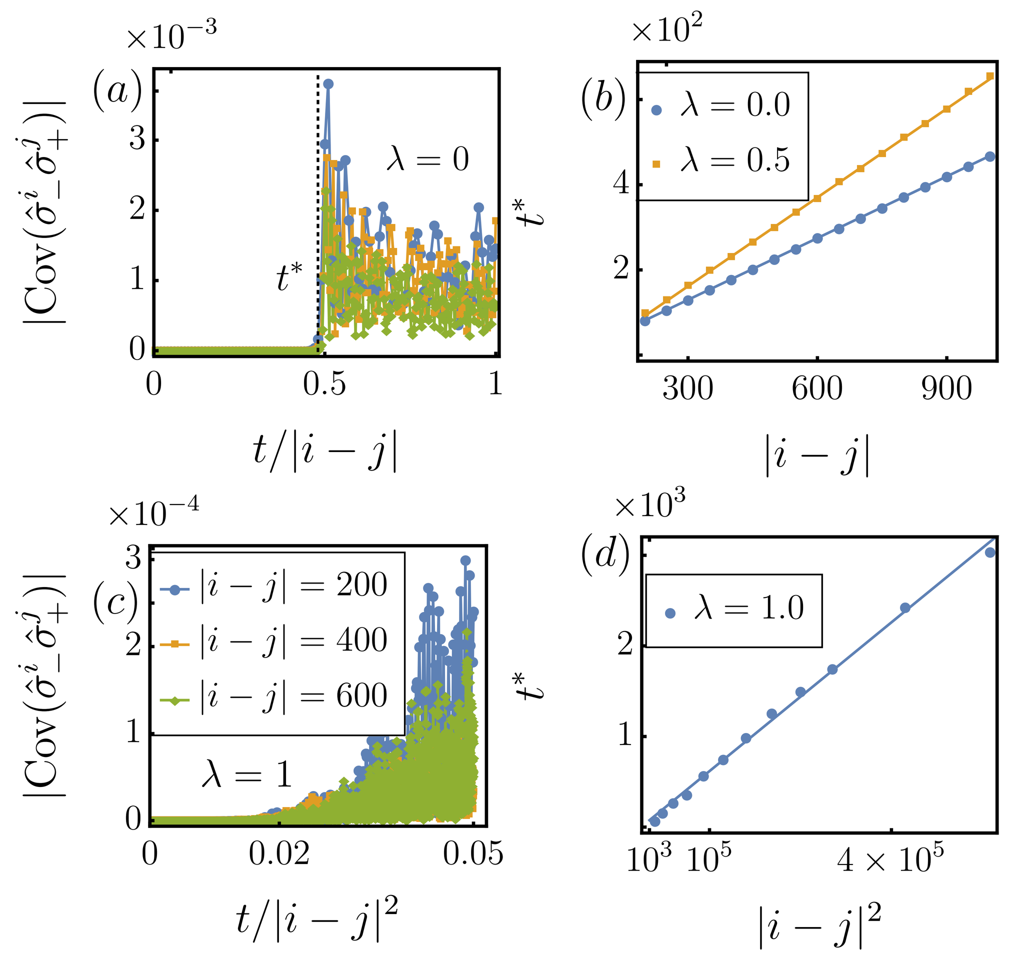

Figure 4: Dynamics of correlation between two qubits coupled to sites i and j of the regular AAH model, as measured by the magnitude of the covariance, is shown for the delocalized regime ($\lambda < 1$) in (a) and critical regime in (b) ($\lambda = 1$). In both cases the correlations start building in a significant manner after a threshold time t^*. While t^* scales linearly with the separation $|i - j|$ in the delocalized regime as shown in (c), in the critical regime it displays quadratic scaling - see (d). Here, we have fixed $i = N/4$, $j = 3N/4$ varied the system size as $N = 400, 800$ and 1200 respectively to change $|i - j|$.

0.2.2 Prediction of localization length:

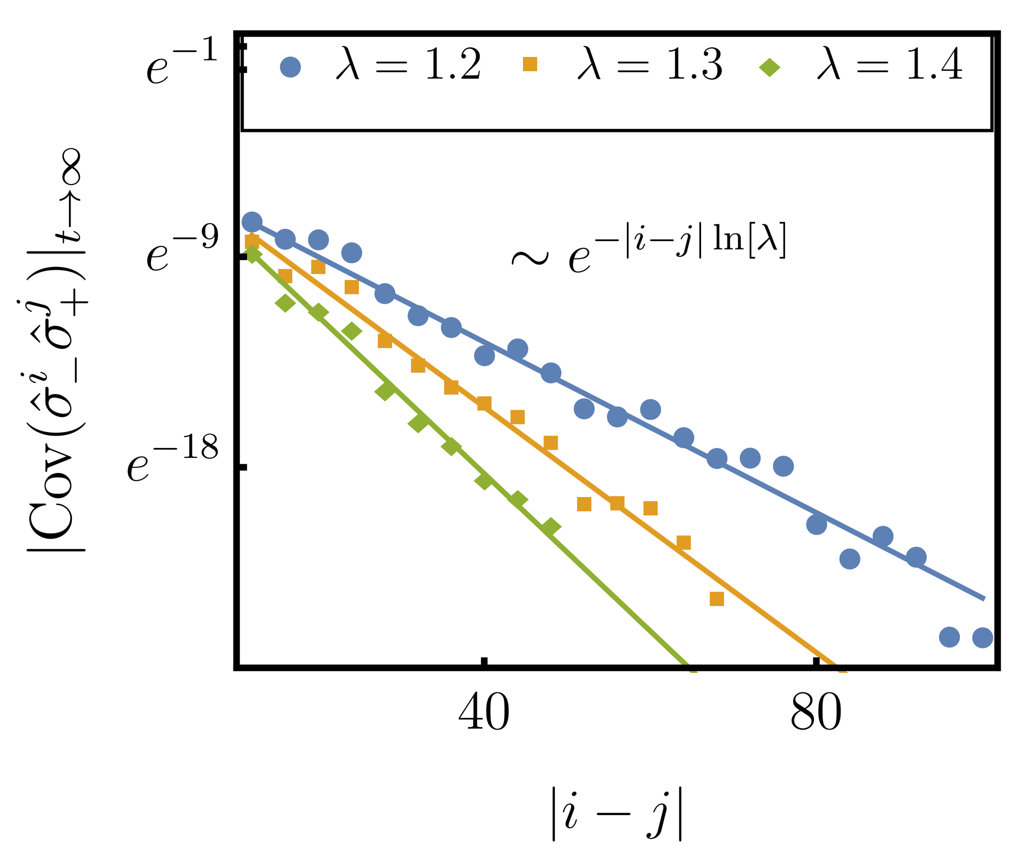

Figure 5: Magnitude of covariance at long times ($\sim 10^4$) is plotted as a function the distance $i - j$ between two qubits in the localized regime of regular AAH model with $\lambda = 1.2$ (blue dots), 1.3 (orange squares) and 1.4 (green squares). Clearly the exponential decay of the covariance agrees with the straight lines which represent an exponential decay of the form $\exp(-|i - j|\ln[\lambda])$ expected from the behavior of localized SPEs.

Conclusions and future questions

- We have readout the nature of single particle properties and isolated system transport properties from the one qubit decoherence dynamics and two qubit correlation dynamics.
- We have also found the localization length using the two-qubit correlation dynamics.
- Can we readout any out-of-equilibrium properties using the qubit probe?

References

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